12. Atom

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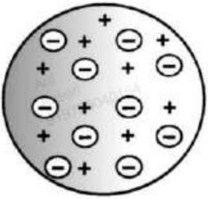
Thomson Model of Atom

According to the Thomson model, every atom consist of a positively charged sphere of radius of the order of 10^{-10}m in which entire mass and positive charge of the atom are uniformly distributed. Inside this sphare, the electrons are embeded like seeds in a watermelon. or like plums in a pudding. The number of electron is such that their negative charge is equal to the positive charge of the atom. Thus the atom is electrically neutral.

Limitations of Thomson Atom model

(i) It could not explain the origin of spectral series of hydrogen and other atoms, observed experimentally.

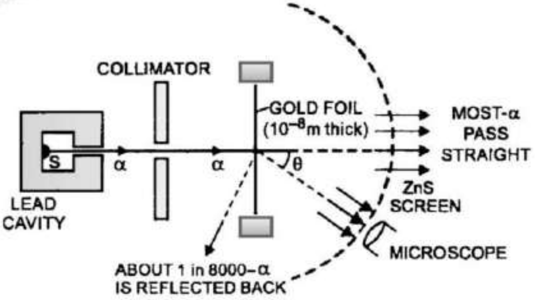
(ii) It could not explain the large angle scattering of α-particles from thin metal foils, as observed by Rutherford.



Rutherford's α-ray Scattering Experiment:

The experimental setup used by Rutherford and his Collaborators, Geiger and Marsden is shown in fig -

S is a piece of radioactive source ($_{83}Bi^{214}$) Contained in a lead cavity. The α- particles emitted by the source are collimated into a narrow beam with the help of a Lead slit (Collimator).

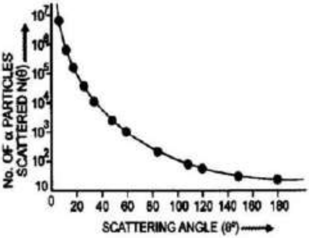


The Collimated beam is allowed to fall on a thin gold foil of thickness of the order of 2.1×10^{-7} m. The α- particles scattered in different directions are observed through a rotatable detector Consisting of a zinc sulphide screen and a microscope.

The α- particles produce bright flashes on the ZnS screen. These are observed in the microscope and counted at different angles from the direction of incidence of the beam. The angle θ of deviation of an α- particle from its original direction is called its scattering angle θ.

Observations -

A graph is plotted between the scattering angle θ and the number of α-particles N, scattered at $\angle\theta$ for a large number of α-particles. We find that -



(i) Most of the α-particles pass straight through the gold foil. It means they do not suffer any collision with gold atoms.

(ii) Only about 0.14% of incident α-particles scatter by more than 1°.

(iii) About one α-particle in every 8000 α particles deflects by more than 90°.

Explanation -

An α-particle is over 7000 more massive than an electron, and in this experiment, α-particle is travelling at a high speed, therefore, very strong force alone could have deflected them through large angles.

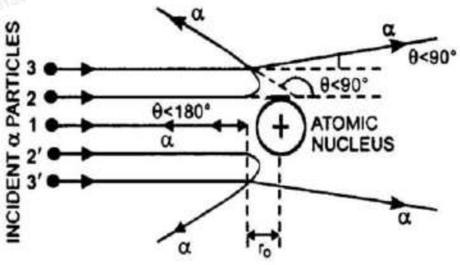
This led Rutherford to postulate that the entire positive charge of the atom must be concentrated in a tiny central core of the atom. This tiny central core of each atom was called atomic nucleus.

An α-particle carries two units of positive charge and has mass of a helium atom. Charge on gold nucleus = Ze, where Z of gold is 79. As gold nucleus is about 50 times heavier than an α-particle, we assume that it would remain stable in the scattering process.

Therefore, the trajectory of α-particle can be computed using Newton's law of motion and Coulomb force of repulsion between α-particle and gold nucleus i.e.

$$\boxed{F = \frac{1}{4\pi\epsilon_0}\frac{(ze)(2e)}{r^2}}$$

where r is the distance of α-particle from the centre of the nucleus.



As shown in fig an α-particle (1) tending to collide with the nucleus, slow down due to repulsive force of the nucleus, finally stops and is then repelled back. This α-particle, therefore retrace its path, scattering through 180°.

The α-particles 2 and 2' tending to hit the nucleus at its periphery, experience strong repulsive force and get scattered through large angles (θ > 90°).

The α-particles 3 and 3', which pass at a distance from the nucleus experience small repulsive forces and get scattered through small angles.

The α-particles which pass at Large distances from the nucleus go almost undeviated.

We can show that the number of α-particles scattered per unit area $N(\theta)$ at scattering angle θ varies inversely as $\sin^4(\frac{\theta}{2})$.

$$\boxed{N(\theta) \propto \frac{1}{\sin^4(\frac{\theta}{2})}}$$

Distance of Closest Approach-

When α-particle is directed towards the nucleus, the Kinetic energy of α-particle goes on decreasing and in turn electrical potential energy goes on increasing due to Coulomb's repulsive force between nucleus and α-particle.

At a certain distance r_0 from the nucleus, K.E. of α-particle reduces to zero. The particle stops and it cannot go closer to the nucleus. It is repelled by the nucleus and therefore, it retrace its path, turning through 180°. This distance r_0 is known as the distance of closest approach.

Electrical potential at distance r_0 due to nucleus

$$\boxed{V = \frac{1}{4\pi\epsilon_0} \times \frac{Ze}{r_0}}$$

Potential Energy of α-particle at distance r_0 from the nucleus = Potential $\times$ Charge

$$\boxed{P.E. = \frac{Ze}{4\pi\epsilon_0 r_0} \times 2e}$$

Kinetic Energy of α-particle of mass m moving with velocity v is

$\Rightarrow$ $\boxed{K.E. = \frac{1}{2}mv^2}$

As at the distance of closest approach -

$$K.E. = P.E.$$

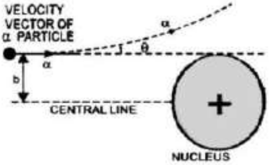
$$\frac{1}{2}mv^2 = \frac{ze \times 2e}{4\pi\epsilon_0 r_0}$$

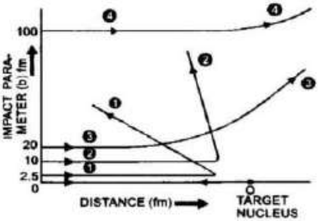
or

$$\boxed{r_0 = \frac{ze \times 2e}{4\pi\epsilon_0\left(\frac{1}{2}mv^2\right)}}$$

Impact Parameter (b)

It is defined as the perpendicular distance of the initial velocity of α-particle from the central line of the nucleus, when the particle is far away from the nucleus.





When the impact parameter is large, an α-particle will deviate through a much smaller angle. However, when impact parameter is small, force experienced is large and the α-particle will scatter through a large angle.

Rutherford calculated analytically the relation between the impact parameter b and scattering angle θ, which is given by-

$$\boxed{b = \frac{1}{4\pi\epsilon_0}\frac{ze^2 \cot \theta/2}{K.E.}} \qquad \left[K.E. = \frac{1}{2}mv^2\right]$$

Rutherford's Atom Model -

(1.) Every atom consist of a tiny central core, called the atomic nucleus in which the entire positive charge and almost entire mass of the atom are concentrated.

(2.) The size of the nucleus is of the order of 10^{-15} m. which is very small as compared to the size of the atom which is of the order of 10^{-10} m.

(3.) The atomic nucleus is a number of electrons. As atom is electrically neutral, the total negative charge of electrons surrounding the nucleus is equal to total positive charge on the nucleus.

(4.) These electrons revolve around the nucleus in various circular orbits. The centripetal force required by electron for revolution is provided by the electrostatic force of attraction between the electrons and the nucleus.

Energy of the electron in orbit

Let F_c = Centripetal force required to Keep a revolving electron in orbit

F_e = electrostatic force of attraction between the revolving electron and the nucleus

then for a dynamically stable orbit in a hydrogen atom $(z=1)$ -

$$F_c = F_e$$

$$\frac{mv^2}{r} = \frac{e \cdot e}{4\pi\epsilon_0 r^2} \Rightarrow mv^2 = \frac{e^2}{4\pi\epsilon_0 r}$$

K.E. of the electron in the orbit $= \frac{1}{2}mv^2$

Hence $$\boxed{K.E. = \frac{e^2}{8\pi\epsilon_0 r}}$$

potential energy of electron in orbit -

$$\boxed{U = \frac{e(-e)}{4\pi\epsilon_0 r} = \frac{-e^2}{4\pi\epsilon_0 r}}$$

$\therefore$ Total energy of electron in hydrogen atom -

$$E = K.E. + U$$

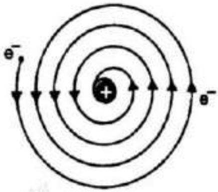
$$E = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r}$$

or $$\boxed{E = -\frac{e^2}{8\pi\epsilon_0 r}}$$

Hence the total energy of electron in orbit of hydrogen atom is negative. Hence, the electron is bound to the nucleus i.e. the electron is not free to leave the orbit around the nucleus.

Limitation of Rutherford Atom Model -

(i) According to the classical EM theory, the revolving electrons must radiate energy in the form of EM waves. As revolving electron loses energy continuously, it must spiral inwards and finally fall into the nucleus, but as matter is stable, we cannot expect the atoms to collapse.



(ii) As the revolving electrons spiral inwards, their angular velocities and hence their frequencies of revolution would change continuously. Therefore, frequency of EM waves emitted must change continuously.

Therefore, atoms should emit continuous spectrum but we observe only a line spectrum.

Bohr Model of Hydrogen Atom-

There are three basic postulates of this model-

(1) Every atom consist of a central core called nucleus, in which entire positive charge and almost entire mass of the atom are concentrated. A suitable number of electrons revolve around the nucleus in circular orbits. The centripetal force required for revolution is provided by the electrostatic force of attraction between the electron and the nucleus.

Centripetal force = Electrostatic force of attraction

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{(ze)(e)}{r^2}$$

$$\therefore \quad \boxed{\frac{mv^2}{r} = \frac{Kze^2}{r^2}} \qquad \left[\begin{array}{l} K = 1/4\pi\epsilon_0 \\ z = 1 \end{array}\right.$$

(2) According to Bohr, electron can revolve only in certain discrete non radiating orbits, called stationary orbits, for which total angular momentum of the revolving electron is an integral multiple of $h/2\pi$, where h is plank's Constant.

Thus the angular momentum of a orbiting electron is quantised.

$$\boxed{mvr = \frac{nh}{2\pi}} \qquad n = 1, 2, 3 \ldots$$

Here n is called principle quantum number.

* The electron, while revolving in such orbits, shall not lose energy i.e. its energy would stay Constant.

(3.) The emission / absorption of energy occurs only when an electron jumps from one of its specified non-radiating orbit to another. The difference in the total energy of electron in the two orbits is absorbed when the electron jumps from an inner to an outer orbit and emitted when electron jumps from outer to the inner orbit.

$$\boxed{h\nu = E_2 - E_1}$$

where ν is the frequency of radiation emitted on jumping from outer to inner orbit of energy E_2 and E_1 respectively.

Radius of Bohr's stationary Orbits-

We Know that for stationary orbits-

$$mvr = \frac{nh}{2\pi} \quad \Rightarrow \quad v = \frac{nh}{2\pi mr}$$

By putting the value of v in $\frac{mv^2}{r} = \frac{Kze^2}{r^2}$

$$\Rightarrow \frac{m}{r} \times \frac{n^2h^2}{4\pi^2m^2r^2} = \frac{Kze^2}{r^2}$$

$$\boxed{r = \frac{n^2h^2}{4\pi^2mKe^2}}$$ for hydrogen atom $Z = 1$

It shows that - $r \propto n^2$

Hence the radius of stationary orbits are in the ratio $1^2 : 2^2 : 3^2 :$ and so on i.e. $1:4:9:\ldots$ Clearly the stationary orbits are not equally spaced.

Velocity of electron in Bohr's Stationary orbit

As we Know that - $\frac{mv^2}{r} = \frac{kze^2}{r^2}$

or $r = \frac{kze^2}{mv^2}$ —— (1)

and also $r = \frac{nh}{2\pi mv}$ —— (2)

$\therefore$ by equation (1) and (2) we get -

$$\frac{kze^2}{mv^2} = \frac{nh}{2\pi mv} \quad \text{or} \quad v = \frac{2\pi kze^2}{nh}$$

for hydrogen atom, $z = 1$ $\boxed{v = \frac{2\pi ke^2}{nh}}$

As $v \propto \frac{1}{n}$, hence the orbital velocity of electron in outer orbits is smaller as compared to its value in the inner orbits.

Frequency of electron in Bohr's Stationary orbit

It is the number of revolutions completed per second by the electron in a stationary orbit, around the nucleus. It is represented by ν.

As $v = r\omega = r(2\pi\nu)$

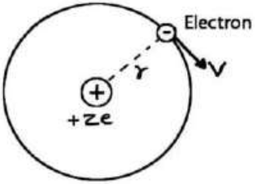
$$\nu = \frac{v}{2\pi r} = \frac{2\pi kze^2}{nh \cdot 2\pi r} = \frac{kze^2}{nhr}$$

$$\boxed{\nu = \frac{kze^2}{nhr}} \Rightarrow \boxed{\nu = \frac{ke^2}{nhr}}$$ $z = 1$ for hydrogen

Total Energy of electron in Bohr's Stationary Orbit

Kinetic energy of electron revolving in a stationary orbit is -

$$K.E. \text{ of electron} = \frac{1}{2}mv^2 = \frac{Kze^2}{2r} \quad \left[\text{by } \frac{mv^2}{r} = \frac{Kze^2}{r^2}\right]$$



Potential energy of electron = Potential × Charge

$$P.E. \text{ of electron} = \frac{Kze}{r} \times (-e) = -\frac{Kze^2}{r}$$

Total energy of electron in the orbit -

$$E = K.E. + P.E.$$

$$E = \frac{1}{2}\frac{Kze^2}{r} - \frac{Kze^2}{r} = -\frac{Kze^2}{2r}$$

by putting $r = \frac{n^2h^2}{4\pi^2 m Kze^2}$ we get -

$$\boxed{E = -\frac{2\pi^2 m K^2 z^2 e^4}{n^2 h^2}}$$

by substituting the standard values, we get -

$$\boxed{E = -\frac{13.6}{n^2} \text{ eV}} \qquad E \propto \frac{1}{n^2}$$

Hence the total energy of electron in a stationary orbit is negative, which means that the electron is bound to the nucleus and is not free to leave it.

* When $n = 1$ then this state of lowest energy of the atom is called ground state. The energy of this state is $E_1 = -13.6$ eV.

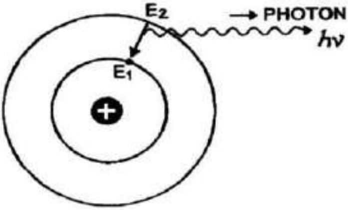
* Therefore, the minimum energy required to free the electron from the ground state of hydrogen atom is 13.6 eV. This is called ionisation energy of hydrogen atom.

* As n increases, the value of negative energy decreases i.e. energy is progressively larger in the outer orbits.

$$\boxed{\text{T.E.} = -\text{K.E.} = \frac{\text{P.E.}}{2}}$$

Origin of Spectral Lines

At room temperature, most of the hydrogen atoms are in ground state. When a hydrogen atom receives energy by processes such as electron collisions or heat, the atom may require sufficient energy to raise the electron to higher energy state.



From these excited states, the electron can fall back to a state of lower energy and emitting a photon of particular energy (= difference in energies of the two states).

Let E_1 and E_2 are the total energy of electron in the inner and outer orbit respectively.

When an electron jumps from an outer to an inner orbit, the energy of radiation emitted is given by–

$$h\nu = E_2 - E_1$$

$$h\nu = \frac{2\pi^2 m K^2 z^2 e^4}{h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{hc}{\lambda} = \frac{2\pi^2 m K^2 z^2 e^4}{h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or $$\frac{1}{\lambda} = \frac{2\pi^2 m K^2 e^4 z^2}{ch^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Now $\frac{1}{\lambda} = \bar{\nu}$, where $\bar{\nu}$ is the wave number of radiation emitted i.e. number of complete wave in unit length. and

$$\frac{2\pi^2 m K^2 e^4}{ch^3} = R \quad \text{(Rydberg Constant)}$$

or $$R = 1.097 \times 10^7 \ m^{-1}$$

Hence $$\boxed{\bar{\nu} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]}$$ For hydrogen $Z = 1$

Above equation is called Rydberg formula for the spectrum of hydrogen atom.

* By above formula it is clear that wavelength/frequencies/wave numbers of radiations emitted by the excited hydrogen atom are not continuous. They have specific values depending upon the values of n_1 and n_2.

* Frequency of radiation emitted when the atom makes transition from the higher energy state (n_2) to the lower energy state (n_1) –

$$\nu = \frac{c}{\lambda} \Rightarrow \boxed{\nu = Rc\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]}$$

Q. Find the ratio of energies of photons produced due to transition of an electron of hydrogen atom from its (a) second permitted energy level to the first level (b) the heighest permitted energy level to the first permitted level.

CBSE 2018

Sol. (a) Energy of Photon released $= E_2 - E_1$

$= -3.4 - (-13.6) = 10.2$ eV

(b) The heighest permitted energy level to the first permitted level $= E_\infty - E_1$

$= 0 - (-13.6) = 13.6$ eV

Ratio of energies of Photon $= \frac{10.2}{13.6} = \frac{3}{4}$

Q. What is the ratio of radius of the orbits corresponding to first excited state and ground state in a hydrogen atom?

CBSE 2021

Sol. For first excited state $n = 2$

Ground state occurs for $n = 1$

$\because \quad r \propto n^2$

Hence $\frac{r_1}{r_2} = \left(\frac{n_1}{n_2}\right)^2 = \left(\frac{2}{1}\right)^2$

$r_1 : r_2 = 4 : 1$

Q. The radius of innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What is the radius of orbit in the second excited state?

CBSE 2022

Sol. The radius of atom whose principle quantum number is n, is given by $r = n^2 r_0$

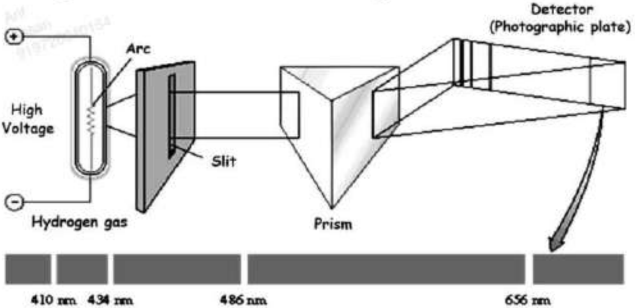
Here r_0 is radius of innermost $(n=1)$ orbit.

For second excited state $n = 3$

$\therefore \quad r = (3)^2 \times 5.3 \times 10^{-11} = 4.77 \times 10^{-10}$ m.

Bohr's Explanation of spectral series of Hydrogen Atom -

When an atomic gas or vapour at low Pressure is excited usually by passing an electric current through it, the gas/vapour emits radiations of certain specific wavelengths only. This kind of spectrum is called "line emission spectrum" and it consists of a few bright lines on a dark background.



When the white light is passed through the same gas/vapour, we observe a bright background crossed by a few dark lines signifying the missing wavelengths or the wavelengths that are absorbed by the gas. They form a "Line absorption spectrum".

It was found that missing wavelengths are the same as the wavelength present in the emission spectrum of the gas/vapour.

Bohr gave his theory of hydrogen atom about spectral series which had observed experimentally by various scientists. Bohr offered a theoretical explanation of these spectral series as follows :

1. Lyman Series –

Bohr postulates that Lyman series is obtained when an electron jumps to the first orbit ($n_1 = 1$) from any outer orbit ($n_2 = 2, 3, 4 \ldots$)

Wave numbers of spectral lines of lyman series were calculated as –

$$\boxed{\bar{\nu} = R\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right]} \qquad (n_2 = 2, 3, 4 \ldots)$$

These values of $\bar{\nu}$ lie in the "ultra violet region" of the spectrum.

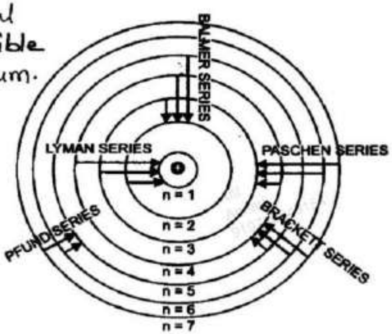
2. Balmer Series –

According to Bohr, Balmer series is obtained when an electron jumps to the second orbit ($n_1 = 2$) from any outer orbit ($n_2 = 3, 4, 5, \ldots$)

Wave number of these spectral lines were calculated as –

$$\boxed{\bar{\nu} = R\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right]} \qquad (n_2 = 3, 4, 5 \ldots)$$

This set of spectral lines lie in the visible part of the spectrum.



3. Paschen Series -

According to Bohr, paschan series is obtained when an electron jumps to the 3rd Orbit ($n_1 = 3$) from any outer orbit ($n_2 = 4, 5, 6, \ldots$).

Bohr calculated the wave number of spectral lines of Paschen series from the relation:

$$\boxed{\bar{\nu} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right]} \quad (n_2 = 4, 5, 6 \ldots)$$

The value of $\bar{\nu}$ lie in the infrared region of the spectrum.

4. Brackett Series -

According to Bohr, Brackett series is obtained when an electron jumps to the 4th orbit ($n_1 = 4$) from any outer orbit ($n_2 = 5, 6, 7 \ldots$)

Wave number $$\boxed{\bar{\nu} = R\left[\frac{1}{4^2} - \frac{1}{n_2^2}\right]} \quad (n_2 = 5, 6, 7, \ldots)$$

Brackett series was discovered in the infrared region of the spectrum.

5. Pfund Series -

According to Bohr Pfund series is obtained when an electron jumps to the 5th orbit ($n_1 = 5$) from any outer orbit ($n_2 = 6, 7, 8, \ldots$)

Wave number $$\boxed{\bar{\nu} = R\left[\frac{1}{5^2} - \frac{1}{n_2^2}\right]} \quad n_2 = 6, 7, 8, \ldots$$

Pfund series was discoverd in the infrared region of the spectrum.

Energy Level Diagram -

A diagram which represents the total energies of electron in different stationary orbits of an atom is called the energy level diagram of that atom.

In this diagram, total energies of electron in various stationary orbits are represented by the parallel horizontal lines drawn according to some suitable energy scale

Total energy of electron in n^{th} orbit of hydrogen atom is -

$$\boxed{E = -\frac{13.6}{n^2} \text{ eV}}$$

putting $n = 1, 2, 3, \ldots$ we get the energies of electrons in various stationary orbits as :

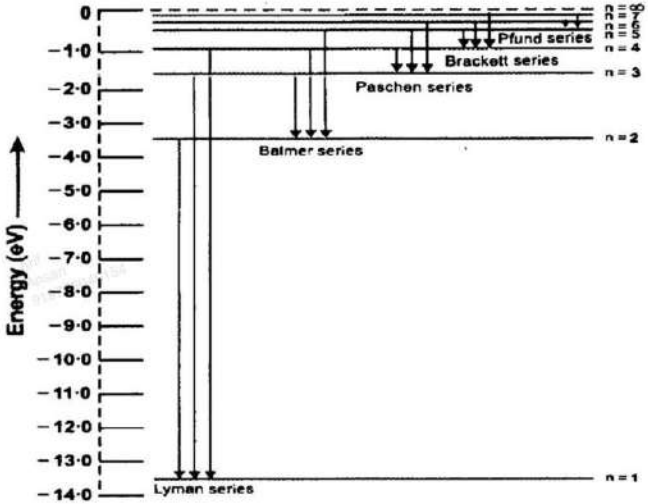
$E_1 = -\frac{13.6}{1^2} = -13.6 \text{ eV}$, $E_2 = \frac{-13.6}{2^2} = -3.4 \text{ eV}$

$E_3 = \frac{-13.6}{3^2} = -1.51 \text{ eV}$, $E_4 = \frac{-13.6}{4^2} = -0.85 \text{ eV}$

$E_5 = \frac{-13.6}{5^2} = -0.54 \text{ eV}$ $E_6 = \frac{-13.6}{6^2} = -0.37 \text{ eV}$

Clearly as n increases E_n becomes less negative until at $n = \infty$, $E_n = 0$.

The energy level diagram is shown in figure for hydrogen atom. The heighest energy state corresponds to $n = \infty$ and has energy $E = 0$ eV. This is the energy of the atom, when the electron is removed ($r = \infty$) from the nucleus and the electron is at rest.



* As n increases energies of the excited state come closer togather.

Q. (CBSE 2019) Calculate the wavelength of H_α line ($n_2 = 3$) in Balmer series of hydrogen atom, given Rydberg Constant $R = 1.097 \times 10^7\ m^{-1}$

Sol. For Balmer series, wavelength is given by-

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right]$$

for H_α line $n_2 = 3$, so that

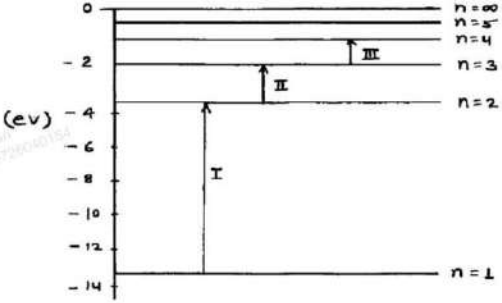
$$\frac{1}{\lambda} = 1.097 \times 10^7 \left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$\frac{1}{\lambda} = 0.1524 \times 10^7$$

$$\lambda = \frac{1}{0.1524 \times 10^7} = 6563\ A^\circ$$

Q. Photons, with a continuous range of frequencies are made to pass through a sample of hydrogen. The transitions shown here, indicate three of the spectral absorption lines in the continuous spectrum.

Delhi 2003



(a) Identify the spectral series of the hydrogen emission spectrum, to which each of these three lines Corresponds.

(b) Which of these lines corresponds to the absorption of radiation of maximum wavelength?

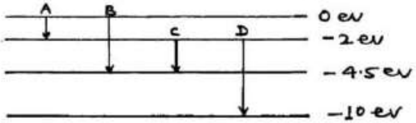
Sol. (a) Ist spectrum represent Lyman series because in this spectrum, electron jumps from $n = 1$ to $n = 2$.

IInd spectrum represent Balmar series because in this spectrum, electron jumps from $n = 2$ to $n = 3$.

IIIrd spectrum represents Paschen series as the electron jumps from $n = 3$ to $n = 4$ in this spectrum.

(b) Spectral line III (Paschen series) Corresponds to the absorption of radiation of maximum wavelength.

Q. CBSE 2022 (a) The energy levels of an atom are shown in fig. Which of them will result in the emission of a photon of wavelength 275 nm?
(b) Which transition corresponds to emission of radiation of maximum wavelength?



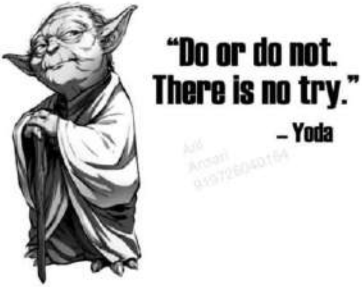
Sol. (a) When $\lambda = 275\,nm = 275 \times 10^{-9}\,m$ then

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{275 \times 10^{-9}}$$

$$E = 4.5\,eV$$

∴ Transition B will result in the emission of photon of $\lambda = 275\,nm$.

(b) Maximum wavelength has minimum energy. Transition A provides energy of 2 eV, which is minimum.



De Broglie's Explanation of Bohr's second postulate of Quantization

The second postulate of Bohr model says that angular momentum of electron orbiting around the nucleus is quantized.

LOUIS DE BROGLIE

$$mvr = \frac{nh}{2\pi}$$

where $n = 1, 2, 3 \ldots$

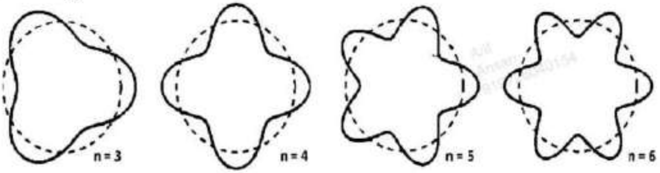
Louis de Broglie explained this puzzle.

According to de Broglie, the electron in its circular orbit, as proposed by Bohr, must be seen as a particle wave.

We know that when a string fixed at two ends is plucked, a large number of wavelengths are excited. But only those waves which have nodes at the two ends from the standing waves and survive.

It means that in a string, standing waves form when total distance travelled by a wave down the string and back is any integer multiple of the wavelengths.

Waves with other wavelengths interfere with themselves upon reflection and their amplitudes vanish quickly.



Hence, according to de Broglie, a stationary orbit is that which contains an integral number of de Broglie waves associated with the revolving electron.

For an electron revolving in n^{th} circular orbit of radius r,

For the permissible orbit, $2\pi r = n\lambda$

According to de Broglie $\lambda = \frac{h}{mv}$

where v is speed of electron revolving in n^{th} orbit

$$\therefore \quad 2\pi r = \frac{nh}{mv}$$

$$\Rightarrow \quad mvr = \frac{nh}{2\pi}$$

$$\boxed{\text{Angular momentum } mvr = n\left(\frac{h}{2\pi}\right)}$$

Limitation of Bohr's Theory -

(1.) This theory is applicable to only simplest atom like hydrogen, with $z=1$. The theory fails in case of atoms of other elements for which $z>1$.

(2.) The theory does not explain why orbits of electrons are taken as circular, while elliptical orbits are also possible.

(3.) Bohr's theory does not say anything about the relative intensities of spectral lines.

(4.) Bohr's theory does not take into account the wave properties of electrons.

Q.1. A single electron orbits around a stationary nucleus of charge +Ze, where Z is a constant and e is the magnitude of the electronic charge. It requires 47.2 eV to excite the electron from second Bohr orbit to the third Bohr orbit. Find:

(a) the value of Z,

(b) the energy required to excite the electron from n = 3 to n = 4,

Sol. (a) transition is $n_1 = 2 \rightarrow n_2 = 3$; $E = 47.2\text{eV}$

we have $\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\text{eV}$

$$\Rightarrow \; 47.2 = 13.6Z^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \Rightarrow Z = 5$$

(b) transition is $n_1 = 3 \rightarrow n_2 = 4$; $E = ?$

we have $\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\text{eV}$

$$\Delta E = 13.6 \times (5^2)\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 16.53 \text{ eV.}$$

Q.2. (a) Find the maximum wavelength λ_0 of light which can ionize a hydrogen atom in its ground state.

(b) Light of wavelength λ_0 is incident on a hydrogen atom which is in its first excited state. Find the kinetic energy of the electron coming out.

Solution : (a) $\frac{hc}{\lambda_0} = 13.6\text{eV}$

$\lambda_0 = 910\ \text{Å}$

(b) K.E. = 10.2 eV

Q.3. (a) Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit.

(b) How many spectral lines are observed in the emission spectrum of the above excited system?

Solution :

(a) The energy in the first orbit $E_1 = Z^2 E_0$ where $E_0 = -13.6eV$ is the energy of a hydrogen atom in ground state. Thus for Li^{++}, $Z = 3$

$E_1 = 9E_0 = 9 \times (-13.6eV)$

The energy in the third orbit is

$$E_3 = \frac{E_1}{n^2} \qquad = \frac{E_1}{9} = -13.6eV$$

Thus, $E_3 - E_1 = 8 \times 13.6eV = 108.8eV$

The wavelength of radiation required to excite Li^{++} from the first orbit to the third orbit is given by

$$\frac{hc}{\lambda} = E_3 - E_1 \quad \text{or} \quad \lambda = \frac{hc}{E_3 - E_1}$$

$$\lambda = \frac{12375}{108.8eV}(\text{Å}) \quad \text{or} \quad \lambda = 114\text{Å}$$

(b) The spectral lines emitted are due to the transition $n = 3 \rightarrow n = 2$, $n = 3 \rightarrow n = 1$ and $n = 2 \rightarrow n = 1$, thus there will be three spectral lines.

Q.4. Find the radius of Li^{++} ions in its ground state assuming Bohr's model to be valid.

Sol. For hydrogen-like ions, the radius of the n^{th} orbit is

$$r_n = \frac{n^2 r_0}{Z}$$

For Li^{++}, $Z = 3$ and in ground state $n = 1$

The radius is $r_1 = \dfrac{1^2 \times 53 \times 10^{-12}}{3} = 18 \times 10^{-12}$ m